Asymmetries, Passive Partial Ownership Holdings, and Product Innovation

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Abstract

We study how asymmetries affect R&D investments, competition, and welfare in markets with passive partial ownership holdings between rival firms. The asymmetries that we consider can arise either because one firm enjoys an initial competitive advantage, or because the passive partial ownership holdings between rivals are unequal. In contrast to previous findings, we find that, due to asymmetries, passive partial ownership holdings can increase total surplus in markets with competition in prices and quality-enhancing R&D with no spillovers. In these markets, our finding suggests that competition authorities should take into account the potential beneficial effects of asymmetric passive partial ownership holdings.

Keywords: partial ownership, minority shareholdings, R&D investments, price competition, competition policy

JEL codes: D43, L11, L40, G34

1 Introduction

There has been a recent surge in passive partial ownership holdings (hereafter PPO) between rival firms which have attracted the attention of competition authorities around the world. An important characteristic of these are the asymmetries between firms, either because one firm enjoys an initial competitive advantage, or because the PPO holdings between rivals are unequal. Our objective is to study, in the presence of PPO, how asymmetries between rivals affect R&D investments, competition in the product market, and welfare. In contrast to previous findings, we find that, due to asymmetries, PPO can increase total surplus in markets with competition in prices and quality-enhancing R&D with no spillovers.

We consider a two-period model with two firms. In the first period, firms invest in R&D to increase product quality. We examine markets where R&D spillovers are negligible due to high

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1These are subject to merger control in the UK, Germany and the US, among others. The European Commission (EC, 2014, 2016) is currently reviewing the issue, specifically in cases where minority shareholdings acquisitions generate a "competitive significant link", which refers to a situation where either of the following conditions is satisfied: a) acquisitions of minority shareholdings in a competitor or vertically related company; b) when either the level of acquired shareholding is around 20% or is greater than 5% but it is accompanied with additional rights.
patent protection. In the second period, firms compete in prices à la Hotelling for the demand of a differentiated product that is purchased by a continuum of consumers. We study how different ownership structures affect equilibrium outcomes, and focus on those in which firms own (small) PPO holdings of rivals with no control rights. Our paper examines how two distinct types of asymmetries affect incentives to invest in R&D, and as a result, firms’ competitive position in the second period. One asymmetry concerns unbalanced financial interests by rivals. We model this asymmetry by supposing that one firm (the acquirer) has a PPO stake in the rival, while the rival (or target firm) does now have a PPO holding in the firm. The other asymmetry is related to situations in which one of the firms enjoys a larger initial competitive advantage since because of brand loyalty or a higher-quality product.

We find that, in markets with symmetric PPO holdings and no competitive advantage by neither firm, an increase in PPO holdings rise equilibrium prices and reduce the total R&D investment. In the first period, each firm is aware that an increase in its price increases the rival’s profit, which can then be partially internalized by the firm since it owns a fraction of the firm’s equity. In addition, because investing in R&D decreases the rival’s profit, and it is costly, firms under-invest in R&D in relation to in markets with no PPO.

We then consider markets with symmetric PPO holdings and in which one firm has an initial competitive advantage. We show that the firm with a competitive disadvantage invests less in R&D with symmetric PPO holdings than with no PPO holdings. This is because increasing R&D is less effective at generating profits, and yet because of symmetric PPO, the firm with disadvantage will be able to appropriate a share of the rival’s profits. The firm with a competitive disadvantage will sell to less than a half of the market. If the competitive advantage is sufficiently large, the firm with a competitive advantage may invest more in R&D with symmetric PPO than with no PPO since the benefit of further increasing its market share outweights the cost of investing in R&D.

With asymmetric PPO holdings and neither firm enjoying an initial competitive advantage, we find that the target firm invests more in R&D than the equivalent firm in a market with no PPO holdings, while the acquirer invests less. The target firm has to over-invest in R&D in order to generate a positive R&D differential which leads to a higher market share, and potentially higher prices, thus augmenting revenues, and profits. By contrast, the acquirer has an incentive to reduce the investment in R&D as asymmetric PPO holdings increase since it appropriates a higher proportion of the target’s profits generated both through the increase in the target’s prices and market share. As a result, the acquirer sells to less than half of the market. Similar results hold when the target firm enjoys an initial competitive advantage. However, when the acquirer has a sufficiently large competitive advantage, then it can then happen that its R&D investment increases as asymmetric PPO holdings increase: the acquirer invests in R&D in order to generate a product of greater quality that will be enjoyed by a larger proportion of consumers.

Our main finding is that asymmetries in markets with PPO may lead to a larger total surplus compared to markets with no PPO. In markets where firms have asymmetric PPO holdings and neither firm enjoys an initial competitive advantage, as PPO holdings increase, the target firm increases its R&D investment but also its market share. This means that aggregate utility
increases since more consumers buy the good of higher product quality generated by the R&D investment in relation to markets with no PPO holdings. Total surplus with asymmetric PPO will be larger than with no PPO if the marginal utility due to R&D is sufficiently large in relation to cost of investing in R&D and to the aggregate transport cost. Nevertheless, consumer surplus is lower with PPO compared to no PPO, while producer surplus is higher with PPO compared to no PPO.

In contrast, total surplus with symmetric PPO and neither firm enjoying a competitive advantage is always lower than in markets with no PPO. This is because firms under-invest in R&D, which implies that a larger proportion of consumers buy a good of lower quality. This decreases the aggregate utility more than the total cost savings due to a lower investment in R&D. However, if one of the firms enjoys a sufficiently large initial competitive advantage then total surplus with symmetric PPO is also larger than total surplus with no PPO. The firm with the initial competitive advantage invests more in R&D than the firm with a disadvantage, thus generating a positive R&D differential, which leads to a higher market share for the firm with the initial competitive advantage. As a result, aggregate utility is larger than in markets with no PPO due to: (1) higher proportion of consumers buying the good with a greater initial gross utility; (2) higher proportion of consumers buying the good of higher quality due to the larger investment in R&D by the firm with the initial competitive advantage. Total surplus with symmetric PPO and an asymmetry in competitive advantage is larger than with no PPO when this increase in aggregate utility outweighs transport and R&D investment costs.

Total surplus may also be larger with PPO than with no PPO when the two types of asymmetries are present. If the target firm enjoys an initial competitive advantage then both asymmetries reinforce each other and the target firm over-invests in R&D. Hence, total surplus is larger with asymmetric PPO than with no PPO if the marginal utility of investing in R&D is large in relation to transport and R&D investment costs. If the acquirer enjoys an initial competitive advantage then it is also possible that total surplus is larger with PPO than with no PPO since the competitive advantage mechanism will mean that it might invest more in R&D compared to no PPO. However, it is less likely that total surplus is greater with PPO than with no PPO in relation to when the target has a competitive advantage.

Our paper is related to three branches of literature. First, it is related to the literature on cooperative R&D and spillovers, which among others includes the seminar articles of d’Aspremont and Jacquemin (1988), Suzumura (1992), Ziss (1994), Kamien et al. (1992), and Leahy and Neary (1997). This literature focuses on cost-reducing R&D investments, while we consider quality-enhancing R&D investments. The second branch of literature examines the effects of passive partial ownership holdings on competition. The seminar articles of Bresnahan and Salop (1986), Reynolds and Snapp (1986), and Allen and Phillips (2000) show that PPO holdings are anti-competitive and result in higher prices. However, in these articles R&D investments are absent. Finally, our paper is related to López and Vives (2016) which consider cost-reducing R&D investment with spillovers, and show that PPO holdings may increase welfare only if markets are not too concentrated, demand is not too convex and moreover spillovers are sufficiently large. In contrast, in the presence of quality-enhancing R&D investments, we show that total

\footnote{These results have also been corroborated by the empirical literature, such as in Azar et al (2015).}
welfare may increase with PPO holdings even if there are no spillovers in the industry.

The paper is organized as follows. Section 2 describes the model. Section 3 finds the equilibrium prices, market shares and R&D investments for different ownership structures. Section 4 conducts welfare analysis. Section 5 discusses the strategic incentives to acquire minority shareholdings. Section 6 concludes.

2 Model

There are two firms, \( i, j = A, B \) with \( i \neq j \), and \( \omega_i \) is firm \( i \)'s share of profits in firm \( j \). The profit of firm \( i \)'s portfolio of investments is

\[
\Pi_i = (1 - \omega_j) \pi_i + \omega_i \pi_j,
\]

where

\[
\pi_i = p_i s_i - \frac{\lambda s_i^2}{2},
\]

where \( s_i \) is firm \( i \)'s market share. We are interested in the case where there are asymmetries between the equity interests of the two firms: \( \omega_A = \omega, \omega_B = 0 \). In order to analyze this question, we consider the benchmark of no PPO \( (\omega_A = \omega_B = 0) \), and study the case of symmetric PPO, where PPO holdings between rivals are equal \( (\omega_A = \omega_B = \omega) \). \(^3\) We restrict that PPO satisfies \( 0 \leq \omega_i, \omega_j < 1/4 \). \(^4\) For a given ownership structure \( (\omega_A; \omega_B) \), we define the total equilibrium R&D investment as

\[
X^*(\omega_A; \omega_B) = x_A^*(\omega_A; \omega_B) + x_B^*(\omega_A; \omega_B).
\]

There is a mass of consumers uniformly distributed along the unit line that have a unit demand. Consumers can purchase the good either from firm \( A \) or firm \( B \), and we assume full participation. We use Hotelling model for this type of demand.

The timing of the model is as follows.

- At \( t = 1 \), each firm chooses an R&D investment, \( x_i \), by maximizing the profits from its portfolio of investments, \( \Pi_i \).
- At \( t = 2 \), given an R&D investment, each firm sets a price, \( p_i \), for a differentiated product, and subsequently consumers purchase from either firm. A consumer that buys from firm \( i \) obtains the following utility

\[
U_i(x_i^t, x_i) - t|q_i - q| - p_i,
\]

where \( t > 0 \) is the product differentiation parameter; \( q \in \{0, 1\} \) is the location of the firm (without loss of generality assume that \( q_A = 0, q_B = 1 \)); and \( q \in [0, 1] \) is the location of a consumer.

\(^3\)Passive partial ownership (PPO) stakes are also often called passive minority financial interests, non-controlling minority shareholdings, or passive cross-ownership.

\(^4\)There is not a commonly agreed threshold about what constitute non-controlling minority shareholdings by competitors. However, competition authorities often inspect the non-controlling minority shareholdings by competitors that are between 15% and 25% (Salop and O'Brien 2000).
Hence, a consumer that is located at \( q \) loses \( t|q_i - q_j| \) units of utility when he purchases the good located \( q_j \). The utility function of a consumer that buys from firm \( i \) is as follows

\[
U_i(x_0^i, x_i) = x_0^i + \rho x_i,
\]

where \( x_0^i \) is the initial gross utility of buying good sold by firm \( i \); consumer’s utility function increases by \( \rho > 0 \) for each unit of R&D invested by firm \( i \). Hence, R&D is of the quality-enhancing type.

Let \( q \) be the consumer that is indifferent between buying \( A \) and \( B \), then:

\[
U_A(x_0^A, x_i) - t|q| - p_A = U_B(x_0^B, x_i) - t|1 - q| - p_B.
\]

Therefore, the market share of firm \( A \) is:

\[
s_A = \frac{1}{2t}((x_0^A - x_0^B) + \rho(x_A - x_B) + (p_B - p_A) + t),
\]

and firm \( B \)’s market share is \( s_B = 1 - s_A \). Without loss of generality, denote \( s_A = s \) and \( s_B = 1 - s \). Define the difference in the initial gross utility from buying from firm \( A \) and firm \( B \) as: \( \Delta_A = x_0^A - x_0^B \), and conversely, \( \Delta_B = x_0^B - x_0^A = -\Delta_A \).

As explained in the introduction, we consider two distinct types of asymmetries.

- Asymmetry due to firm \( i \) enjoying an initial competitive advantage (disadvantage). In our model this translates to \( \Delta_i > 0 \) (\( \Delta_i < 0 \)), which leads to a positive (negative) difference in initial gross utility derived from buying from firm \( i \).

- Asymmetry due to unequal PPO holdings between firms. We model this asymmetry as \( \omega_i = \omega \) and \( \omega_j = 0 \), for \( i \neq j \).

The equilibrium concept used is the Subgame Perfect Equilibrium. A firm’s strategy consists of an investment in R&D and a subsequent pricing strategy which is based on the R&D investment. In terms of welfare, we consider the second-best welfare benchmark such that the planner chooses the R&D that maximizes total surplus, taking as given the equilibrium prices set by the two firms in the second stage. We also compare total surplus at the equilibrium allocation for different ownership structures.

### 3 Equilibrium prices, market shares and R&D Investments

#### 3.1 Benchmark models: No PPO and symmetric PPO

We study two benchmark models with respect to the ownership structure: markets with symmetric PPO (\( \omega_A = \omega_B = \omega \)), and markets with no PPO (\( \omega = 0 \)). We also allow asymmetries in initial competitive advantage to be present, i.e. \( \Delta_i \neq 0 \).

We first find the equilibrium of the model for symmetric PPO. Profits of firms \( i, j = A, B \) with \( i \neq j \) are:
\[ \Pi_i(\omega; \omega) = (1 - \omega)\pi_i + \omega \pi_j, \]

where \( \pi_i = p_i s_i - \frac{\lambda s_i^2}{2} \) and \( s_i = \frac{1}{2t}(\Delta_i + \rho(x_i - x_j) + (p_j - p_i) + t) \). We solve by backward induction and find the optimal prices at \( t = 2 \). The first order condition for each firm \( i \) satisfies \( \frac{\partial \Pi_i}{\partial p_i} = 0 \). Hence, the best response price equations for firms \( i, j = A, B \) are such that for \( i \neq j \):

\[
p_i(\omega; \omega) = \frac{(1 - \omega)\Delta_i + (1 - \omega)p(x_i - x_j) + p_j + (1 - \omega)t}{2(1 - \omega)}.
\]

For given R&D investments, we observe that the price best-reply function is upward sloping (i.e. competition at \( t = 2 \) is of the strategic complements type). Solving for the fixed point, we obtain the prices as functions of R&D investments at \( t = 1 \):

\[
p_i(\omega; \omega) = (1 - \omega) \frac{\Delta_i(1 - 2\omega) + t(3 - 2\omega) + \rho(x_i - x_j)(1 - 2\omega)}{(2\omega - 3)(2\omega - 1)}.
\]

Substituting for these optimal prices we find that the position of the indifferent consumer is: \( s_i = \frac{1}{2t}(3t + \Delta_i + \rho(x_i - x_j)) \). We now solve for the equilibrium R&D investments at \( t = 1 \). The first order condition for firm \( i \) satisfies: \( \frac{\partial x_i}{\partial x_i} = 0 \). The second order condition requires that \( \lambda t (2\omega - 3)^2 - \rho^2 > 0 \), and the stability of the equilibrium that \( \lambda t (2\omega - 3)^2 - 2\rho^2 > 0 \). As a result, we obtain the best response functions for R&D of each firm:

\[
x_i = \rho \left( \frac{\Delta_i - t(1 - \omega)(2\omega - 3)}{(\lambda t (2\omega - 3)^2 - \rho^2)} - \frac{\rho}{\lambda t (2\omega - 3)^2 - \rho^2} x_j \right).
\]

We note that R&D investment at \( t = 1 \) is of strategic substitutes type.

The strategic effect can be shown to be negative.\(^5\) It follows that, at the interior equilibrium, the direct effect is positive, in which case firm \( i \) under-invests in relation to the direct effect. Recall that the sign of the strategic effect depends on whether R&D investment makes firm \( i \) tough or soft, and on whether second period actions are strategic substitutes or complements. R&D investment makes firm \( i \) tough since \( \frac{\partial x_i(x, y, x_j)}{\partial x_i} < 0 \). Furthermore, second period prices are strategic complements. As a result, the strategic effect is necessarily negative. That is, investment in R&D makes firms tough, which combined with strategic complementarity in the second period yield a negative strategic effect, which results in under-investment in R&D. Firms under-invest in R&D so as not to trigger an aggressive response from the rival firm, also known as a "puppy dog" strategy (Fudenberg and Tirole, 1984).

We now solve for the fixed point. The following proposition states the equilibrium outcomes with symmetric PPO.

**Proposition 1** If the second order and stability conditions are satisfied, i.e. \( \lambda t (2\omega - 3)^2 - \rho^2 > 0 \), for each firm \( i \), at equilibrium

\[
x_i^*(\omega; \omega) = \frac{\rho}{\lambda (3 - 2\omega)} \left( \frac{\lambda \Delta_i(3 - 2\omega)}{t \lambda (2\omega - 3)^2 - 2\rho^2} + (1 - \omega) \right),
\]

\(^5\)The following are satisfied: \( \frac{\partial x_i}{\partial x_i} \frac{\partial^2 x_i}{\partial x_i^2} < 0 \); \( \frac{\partial x_i}{\partial x_i} > 0 \).
with \( X^*(\omega; \omega) = \sum_{i} x_i^*(\omega; \omega) + x_B^*(\omega; \omega) = \frac{2\rho(1-\omega)}{\lambda(3-2\omega)}, \)

\[
p_i^*(\omega; \omega) = \frac{(1-\omega)}{(3-2\omega)(1-2\omega)} \left( t(3-2\omega) + \Delta_i(1-2\omega) \left( 1 + \frac{2\rho^2}{(t\lambda (2\omega - 3)^2 - 2\rho^2)} \right) \right), \tag{8}
\]

and

\[
s_i^*(\omega; \omega) = \frac{1}{2} \left( 1 + \frac{\lambda\Delta_i(3-2\omega)}{t\lambda (2\omega - 3)^2 - 2\rho^2} \right). \tag{9}
\]

The special case of no PPO can be obtained from Proposition 1 by setting \( \omega = 0. \)

**COROLLARY 1** (no PPO). If the second order and stability conditions are satisfied, i.e. \( 9\lambda t - \rho^2 > 0, \) for each firm \( i, \) at equilibrium

\[
x_i^*(0; 0) = \frac{\rho}{3\lambda} \left( 1 + \frac{3\lambda\Delta_i}{(9t\lambda - 2\rho^2)} \right),
\]

and \( X^*(0; 0) = x_A^*(0; 0) + x_B^*(0; 0) = \frac{2\rho}{3\lambda}. \)

\[
p_i^*(0; 0) = t \left( 1 + \frac{3\lambda\Delta_i}{9t\lambda - 2\rho^2} \right), \tag{10}
\]

and

\[
s_i^*(0; 0) = \frac{1}{2} \left( 1 + \frac{3\lambda\Delta_i}{9t\lambda - 2\rho^2} \right). \tag{11}
\]

In markets with symmetric PPO and no competitive advantage by neither firm, equilibrium prices are higher than with no PPO since competition is softer in the second stage, and as a result firms invest less in R&D than if firms had no financial links. The reasoning is as follows. With symmetric PPO, firm \( i \) at \( t = 1 \) takes into account that at \( t = 2 \) an increase in \( p_i \) increases firm \( j \)'s profit, which can be partially internalized by firm \( i \) since it owns a fraction \( \omega \) of firm \( j. \)

In addition, due to symmetry and no initial competitive advantage, each firm will still sell to half of the market (since there is full participation). Because investing in R&D decreases the rival’s profit, and it is costly, firms under-invest in R&D in relation to markets with no PPO.

For symmetric PPO, Table 1 describes the comparisons of equilibrium outcomes with symmetric PPO in relation to markets with no PPO for various signs of \( \Delta_i. \)

<table>
<thead>
<tr>
<th>sign(...)</th>
<th>( \Delta_i = 0 )</th>
<th>( \Delta_i &lt; 0 )</th>
<th>( \Delta_i &gt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i^<em>(\omega; \omega) - x_i^</em>(0; 0) )</td>
<td>( - )</td>
<td>( - )</td>
<td>( \text{sign} \left{ \Delta_i - \frac{(9\lambda - 2\rho^2)(t(2\omega - 3)^2 - 2\rho^2)}{12t\lambda(3-2\omega)(3-\omega)} \right} )</td>
</tr>
<tr>
<td>( X^<em>(\omega; \omega) - X^</em>(0; 0) )</td>
<td>( - )</td>
<td>( - )</td>
<td>( - )</td>
</tr>
<tr>
<td>( p_i^<em>(\omega; \omega) - p_i^</em>(0; 0) )</td>
<td>( + )</td>
<td>( \text{sign} \left{ \frac{(9\lambda - 2\rho^2)(t(2\omega - 3)^2 - 2\rho^2)}{(1-2\omega)(1-\omega)} - \Delta_i \left( 3t\lambda(3 - 2\omega) - 2\rho^2(5 - 2\omega) \right) \right} )</td>
<td></td>
</tr>
<tr>
<td>( s_i^<em>(\omega; \omega) - s_i^</em>(0; 0) )</td>
<td>( 0 )</td>
<td>( - )</td>
<td>( + )</td>
</tr>
</tbody>
</table>

Suppose that firm \( i \) has an initial competitive disadvantage, i.e. \( \Delta_i < 0. \) With symmetric PPO, for given levels of R&D and prices, the market share of firm \( j \) will be larger than the
market share of firm $i$. More consumers will find it optimal to purchase from firm $j$ because a larger proportion derive a greater initial gross utility by purchasing from $j$. As a result, firm $j$ will be able to charge higher prices than firm $i$, and generally the equilibrium prices of both firms increase with $\omega$ because of softer competition. Even when $\omega = 0$, firm $i$ has a lower level of R&D investment than firm $j$. In addition, firm $i$ invests less in R&D with symmetric PPO compared to no PPO since it is more costly for this firm to gain market share compared to firm $j$. With symmetric PPO, firm $i$ can take advantage of the R&D investment by firm $j$, which is more effective at generating profits. The combined effect is that firm $i$ has a lower market share than firm $j$.

Suppose now that firm $i$ has a competitive advantage, i.e. $\Delta_i > 0$. The argument is the opposite to the one just described in the previous paragraph, except from the sign of how R&D investment with PPO holdings compares to no PPO holdings since it depends on the size of the competitive advantage. If $\Delta_i$ is large then firm $i$ may invest more in R&D than when there are no PPO holdings. This is because the positive R&D differential between firms lead to a larger differential in prices at $t = 2$, which translate in a larger market share for firm $i$, and higher corresponding profits. In contrast, if $\Delta_i$ is sufficiently small, then the cost of increasing R&D outweighs the benefit derived from the potential increase in revenues. Hence, if $\Delta_i$ is sufficiently small then firm $i$ invests less in R&D with symmetric PPO than with no PPO holdings.

### 3.2 Asymmetric PPO

We consider the case when PPO holdings between rivals are unbalanced, and model it by assuming that $\omega_A = \omega, \omega_B = 0$. This formulation captures the situation in which the acquirer (A) has a PPO holding of the target firm (B), while the target firm does not have any PPO holdings of the acquirer. Profits of firms $A$ and $B$, are

$$\Pi_A(\omega; 0) = \pi_A + \omega \pi_B = p_A s - \frac{\lambda x_A^2}{2} + \omega p_B(1 - s) - \frac{\omega \lambda x_B^2}{2}, \quad (12)$$

$$\Pi_B(\omega; 0) = (1 - \omega) \pi_B = (1 - \omega)\left( p_B(1 - s) - \frac{\lambda x_B^2}{2} \right). \quad (13)$$

Next Proposition states the equilibrium with asymmetric PPO holdings.

**PROPOSITION 2** If the second order and stability conditions are satisfied, i.e. $\lambda t(3 - \omega)^2 - \rho^2 > 0$, at equilibrium

$$x_A^*(\omega; 0) = \rho \left( \frac{\lambda \Delta_A(3 - \omega) - \rho^2(2 - \omega) + t \lambda (3 - \omega)(\omega^2 - 5\omega + 3)}{\lambda (3 - \omega)(t \lambda (3 - \omega)^2 - 2\rho^2)} \right), \quad (14)$$

$$x_B^*(\omega; 0) = \rho \left( \frac{\lambda (3 - \omega)(3t - \Delta_A) - \rho^2(2 - \omega)}{\lambda (3 - \omega)(t \lambda (3 - \omega)^2 - 2\rho^2)} \right). \quad (15)$$
and \( X^*(\omega; 0) = x_A^*(\omega; 0) + x_B^*(\omega; 0) = \frac{\rho(2-\omega)}{\lambda(3-\omega)} \),

\[
p_A^*(\omega; 0) = t \left( \frac{\lambda \Delta_A (3-\omega)(1-\omega) + \rho^2 (\omega^2 - 3\omega - 2) + (3-\omega)(3+\omega) \mu t}{t(3-\omega)^2 - 2\rho^2} \right),
\]

(16)

\[
p_B^*(\omega; 0) = t \left( \frac{- (3-\omega) \lambda \Delta_A - (2-\omega) \rho^2 + 3 t \lambda (3-\omega)}{t(3-\omega)^2 - 2\rho^2} \right),
\]

(17)

and

\[
s_A^*(\omega; 0) = \frac{1}{2} \left( \frac{\lambda \Delta_A (3-\omega) - (2+\omega) \rho^2 - t \lambda (2\omega - 3)(3-\omega)}{t(3-\omega)^2 - 2\rho^2} \right),
\]

(18)

while \( s_B^*(\omega; 0) = 1 - s_A^*(\omega; 0) \).

Suppose that neither the target \((B)\) nor the acquirer \((A)\) enjoy an initial competitive advantage. In the second stage, for given R&D investments, the acquirer takes into account that it internalizes a share \(\omega\) of the rival’s profit, while the target firm only obtains a share \(1-\omega\) of its own profits. For given R&D investments, an increase in \(p_A\) by one unit triggers an increase in \(p_B\) by \(\frac{1}{2}\), while an increase in \(p_B\) by one unit triggers an increase in \(p_A\) by \(\frac{1+\omega}{2}\). Because of the asymmetry in PPO holdings, prices at \(t = 2\) will be more moderate in relation to the symmetric PPO. At \(t = 1\), the target firm augments R&D as PPO holdings increase as a means of generate a positive R&D differential which leads to a higher market share, and potentially higher prices, thus increasing revenues and profits. By contrast, the acquirer has an incentive to reduce the investment in R&D as \(\omega\) increases since it appropriates a proportion \(\omega\) of the surplus generated by target firm both through the increase in prices and market share. In addition, an increase in the R&D investment by the acquirer decreases the target’s profits.

Table 2 shows the comparison of equilibrium outcomes with asymmetric PPO with respect to the no PPO benchmark.

<table>
<thead>
<tr>
<th>(\text{sign}(...))</th>
<th>(\Delta_i = 0)</th>
<th>(\Delta_A &lt; 0, \Delta_B &gt; 0)</th>
<th>(\Delta_A &gt; 0, \Delta_B &lt; 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_A^<em>(\omega; 0) - x_B^</em>(\omega; 0))</td>
<td>(-)</td>
<td>(-)</td>
<td>(\text{sign} \left{ \frac{(3-\omega)(9-2\omega) t \lambda - \rho^2}{3 t \lambda^2 (3-\omega)(6-\omega)} \right} \Delta_A )</td>
</tr>
<tr>
<td>(x_B^<em>(\omega; 0) - x_A^</em>(\omega; 0))</td>
<td>(\text{sign} { 2 t \lambda - \rho^2 } )</td>
<td>(\text{sign} { (2 t \lambda - \rho^2)(3-\omega) - 2 \lambda \Delta_A \frac{3 t \lambda (3-\omega) - \rho^2 (4-\omega)}{9 t \lambda - 2 \rho^2} } )</td>
<td></td>
</tr>
<tr>
<td>(X^<em>(\omega; 0) - X^</em>(0; 0))</td>
<td>(\text{sign} { 2 t \lambda - \rho^2 } )</td>
<td>(\text{sign} { (2 t \lambda - \rho^2)(3-\omega) - 2 \lambda \Delta_A \frac{3 t \lambda (3-\omega) - \rho^2 (4-\omega)}{9 t \lambda - 2 \rho^2} } )</td>
<td></td>
</tr>
<tr>
<td>(p_A^<em>(\omega; 0) - p_A^</em>(0; 0))</td>
<td>(\text{sign} { 2 t \lambda - \rho^2 } )</td>
<td>(\text{sign} { (2 t \lambda - \rho^2)(3-\omega) - 2 \lambda \Delta_A \frac{3 t \lambda (3-\omega) - \rho^2 (4-\omega)}{9 t \lambda - 2 \rho^2} } )</td>
<td></td>
</tr>
<tr>
<td>(p_B^<em>(\omega; 0) - p_B^</em>(0; 0))</td>
<td>(\text{sign} { 2 t \lambda - \rho^2 } )</td>
<td>(\text{sign} { (2 t \lambda - \rho^2)(3-\omega) - 2 \lambda \Delta_A \frac{3 t \lambda (3-\omega) - \rho^2 (4-\omega)}{9 t \lambda - 2 \rho^2} } )</td>
<td></td>
</tr>
<tr>
<td>(s_A^<em>(\omega; 0) - s_A^</em>(0; 0))</td>
<td>(-)</td>
<td>(-)</td>
<td>(\text{sign} \left{ \frac{\lambda - \omega \rho^2 + t \lambda (3-\omega)}{3 \lambda (3-\omega)} \right} \Delta_A )</td>
</tr>
<tr>
<td>(s_B^<em>(\omega; 0) - s_B^</em>(0; 0))</td>
<td>(\text{sign} { 2 t \lambda - \rho^2 } )</td>
<td>(\text{sign} { (2 t \lambda - \rho^2)(3-\omega) - 2 \lambda \Delta_A \frac{3 t \lambda (3-\omega) - \rho^2 (4-\omega)}{9 t \lambda - 2 \rho^2} } )</td>
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</table>

Table 2 shows that with asymmetric PPO holdings and \(\Delta_A \leq 0\), the target firms invests more in R&D compared to an equivalent firm in a market with no PPO holdings, while the
acquirer invests less. In fact, we can show that if $\Delta_A = 0$ then the following relationship holds: $x_B(\omega; 0) > x_A^1(0; 0) > x_A^2(\omega; 0) > x_A(\omega; 0)$. However, when the acquirer has a sufficiently large competitive advantage, i.e. $\Delta_A > 0$, it can then happen that the acquirer increases its R&D investment as asymmetric PPO holdings increase. This is because the acquirer starts with an advantage in market share and quality, and as a result, it can charge higher prices. Because the target firm has a small market share it is more effective that the acquirer invests in R&D in order to generate a product of greater quality that will be enjoyed by a larger proportion of consumers. Overall, the sum of R&D investments by both firms is smaller with asymmetric PPO holdings than with no PPO holdings.

We also observe that the price of the target firm is always larger with asymmetric PPO holdings than with no PPO holdings irrespective of the sign of $\Delta_A$. However, the equilibrium price of the acquirer may be larger or smaller than an equivalent firm with no PPO holdings. If $\Delta_A = 0$ and $\rho^2$ is large in relation to $\tau\lambda$ then consumers give a high value to quality and generally prefer to buy the good produced by the target firm since it invests more in R&D, and hence its product is of higher quality. The acquirer then reduces prices in order to be competitive, and not further reduce its market share. In fact we can show that $\text{sign}(p_A^*(\omega; 0) - p_B^*(\omega; 0)) = \text{sign}(\tau\lambda(3 - \omega) - \rho^2(4 - \omega))$. Consequently, if $\Delta_A \leq 0$ then the market share of the target (acquirer) firm will be larger (smaller) with asymmetric PPO holdings than with no PPO holdings. If the acquirer enjoys a sufficiently large competitive advantage (i.e. if $\Delta_A$ is positive and sufficiently large) then market share of the acquirer will be larger for some $\omega > 0$ in relation to when $\omega = 0$, until the target firm’s proposal is more attractive.

For an illustration of the comparison between equilibrium outcomes for different ownership structures, see Figures 1 and 2.

### 4 Welfare analysis

For a given ownership structure $(\omega_A; \omega_B)$, we define producer surplus, consumer surplus and total surplus. Producer surplus is

$$PS(\omega_A; \omega_B) = \Pi_A(\omega_A; \omega_B) + \Pi_B(\omega_A; \omega_B) = p_A s + p_B (1 - s) - \frac{\lambda}{2} (x_A^2 + x_B^2),$$

where $s_A = s = \frac{1}{\tau}(x_0^A - x_0^B) + \rho(x_A - x_B) + (p_B - p_A) + \tau$. We notice that producer surplus is equal to the total revenue minus the total cost of investing in R&D.

Consumer surplus is equal to

$$CS(\omega_A; \omega_B) = \int_0^s (x_0^A + \rho x_A - ty - p_A)dy + \int_s^1 (x_0^B + \rho x_B - (1 - y) - p_B)dy,$$

which is the sum of consumers’ utility from buying either of the firms minus the total payment transferred to each of the firms and minus the total costs due to product differentiation.

Total surplus is $TS(\omega_A; \omega_B) = CS(\omega_A; \omega_B) + PS(\omega_A; \omega_B)$, and thus equals
Figure 1: Equilibrium outcomes with respect to PPO.

Figure 2: Equilibrium outcomes with respect to output.
\[ TS(\omega_A; \omega_B) = (x_A^0s + x_B^0(1 - s)) + \rho(x_As + x_B(1 - s)) - \frac{t}{2}(s^2 + (1 - s)^2) - \frac{\lambda}{2}(x_A^2 + x_B^2). \] (21)

The next Proposition compares total surplus at the equilibrium allocation, \( TS^*(\omega_A; \omega_B) \), for different ownership structures.

**PROPOSITION 3** Suppose that \( \Delta_i = 0 \). It always holds that

\[ TS^*(0; 0) > TS^*(\omega; \omega). \]

If \( \rho \) is sufficiently large and \( t\lambda \) sufficiently small then

\[ TS^*(\omega; 0) > TS^*(0; 0) > TS^*(\omega; \omega), \]

while if \( \rho \) is sufficiently small and \( t\lambda \) sufficiently large then

\[ TS^*(0; 0) > TS^*(\omega; \omega) > TS^*(\omega; 0) \]

for some \( \omega > 0 \).

The Proposition shows that in markets with no asymmetries due to an initial competitive advantage (\( \Delta_i = 0 \)), total surplus with symmetric PPO is always lower than in markets without PPO. As PPO increases both firms steeply increases prices and under-invest in R&D in relation to markets with no PPO. Yet both firms serve half of the market. Due to PPO, consumer surplus always decreases more than the increase in producer surplus. This is because the under-investment in R&D causes a larger decrease in aggregate utility (since a larger proportion of consumers buy a good of lower quality) which is not compensated by the cost-savings of both firms due to a lower investment in R&D.

The surprising finding is that total surplus with asymmetric PPO may be larger than total surplus with no PPO if the marginal utility due to R&D (\( \rho \)) is sufficiently large in relation to product of the R&D cost parameter, \( \lambda \), and the transportation cost associated with product differentiation, \( t \). This is because, as PPO holdings increase, the target firm increases its R&D investment but also its market share. This leads to a substantial increase in aggregate utility due to more consumers buying a good of higher quality, generated by the R&D investment, in relation to when there are no PPO holdings. When \( \rho \) is sufficiently large in relation to \( t\lambda \), this beneficial allocation effect outweights the increase in costs due to R&D and transport, and hence total surplus is larger with asymmetric PPO than in markets with no PPO. However, we find that due to PPO holdings, consumers are always worse off while producers are always better off. This is due to the effects of prices on both consumer and producer surplus, which is eliminated in total surplus since it is just a transfer from consumers to producers. In contrast, when \( \rho \) is sufficiently small in relation to \( t\lambda \), for some large \( \omega \) total surplus with asymmetric PPO is even smaller than with symmetric PPO since the joint costs of the over-investment in R&D and transport do not compensate the positive increase in aggregate utility.
The next proposition studies the comparison of total surplus with symmetric PPO and no PPO when one of the firms enjoys an initial competitive advantage.

**PROPOSITION 4** If \( \Delta_i \neq 0 \) then

\[
\text{sign} \{ TS^*(\omega; \omega) - TS^*(0; 0) \} = \text{sign} \{ \Delta_i^2 \chi - \zeta \},
\]

where

\[
\chi \equiv \left( 4t \lambda \rho^2 (4 \omega^3 + 12 \omega^2 - 54 \omega + 27) + 9t^2 \lambda^2 (6 - 5 \omega) (2 \omega - 3)^2 - 4 \rho^4 (4 \omega^2 - 13 \omega + 12) \right) > 0 \text{ for the range of } \omega \text{ considered (i.e. } 0 < \omega \leq \frac{3}{4} \text{), and } \zeta \equiv \frac{\rho^2 (3-\omega)(9t\lambda-2\rho^2)(t\lambda(2\omega-3)-2\rho^2)^2}{9t\lambda(2\omega-3)^2} > 0.
\]

If either of the firms has a sufficiently large initial competitive advantage, i.e. \( \Delta_i^2 > \frac{\zeta}{\chi} \), then total surplus with symmetric PPO is larger than with no PPO. Without loss of generality suppose that \( \Delta_i > 0 \). Firm \( i \) invests more in R&D than firm \( j \), thus generating a positive R&D differential, which leads to a higher market share for firm \( i \). As a result, aggregate utility with PPO is larger than with no PPO due to: (1) higher proportion of consumers buying the good with a greater initial gross utility; (2) higher proportion of consumers buying the good of higher quality generated by R&D investment of firm \( i \). Total surplus with symmetric PPO is larger than with no PPO when this increase in aggregate utility outweights the transport and R&D investment costs.

We have conducted simulations to analyze the comparison of total surplus with symmetric, asymmetric PPO and with no PPO, which are displayed in the Figure 3.

The previous figures show the following: (i) When the target firm enjoys an initial competitive advantage (\( \Delta_B > 0 \)) and \( \rho \) is sufficiently large in relation to \( t \lambda \) then total surplus with asymmetric PPO is larger than with no PPO. This is because the mechanism described in Proposition 4 is reinforced since the target firm invests more in R&D, and its market share is greater than if \( \Delta_B = 0 \). In fact, the level of \( \rho \) that is required for total surplus to be higher with asymmetric PPO than with no PPO is smaller as \( \Delta_B \) becomes larger. (ii) When the acquirer enjoys an initial competitive advantage (\( \Delta_A > 0 \)), the level of \( \rho \) that is required for total surplus to be higher with asymmetric PPO than with no PPO increases as \( \Delta_A \) becomes larger. This is because when \( \omega = 0 \), R&D investment, price and market share are larger for the acquirer than the target firm. As asymmetric PPO holdings increase, the R&D investment, price and market share of the acquirer decrease, while those for the target firm increase. For a given level of \( \rho \), aggregate utility is lower than if \( \Delta_A = 0 \) since on average consumers buy a good of a lower initial gross utility and of a lower quality due to the R&D investment by firms, which does not compensate the aggregate transport and R&D costs.

5 **Strategic Incentives to Acquire Minority Shareholdings**

In the previous sections we have considered exogenous ownership structures. In this section, we examine the strategic incentives to acquire minority shareholdings.\(^6\) Let us consider the case

\(^6\)This issue has been previously analyzed by Flath (1991).
Figure 3: Total surplus with symmetric and asymmetric PPO.

of asymmetric shareholdings: \( \Pi_A = \pi_A + \omega \pi_B \) and \( \Pi_B = (1 - \omega)\pi_B \), and include a stage 0 in which the acquirer, \( A \), decides the PPO holdings to invest in the target, \( \omega \). If the market is efficient then firm \( A \) will pay for the equity \( \omega \) exactly the amount \( \omega \pi_B \). Therefore, the net profit is \( \phi_A = \Pi_A - \omega \pi_B = \pi_A \), and firm \( A \) will acquire \( \omega \) from \( B \) only if as a result of the acquisition its operating profit (\( \pi_A \)) increases. Second-period equilibrium yields \( p_i^*(x_i, x_j) \), \( p_j^*(x_i, x_j) \), while first-period equilibrium gives \( x_i^*(\omega) \) and \( x_j^*(\omega) \), which replaced into the former expressions gives \( p_i^*(\omega) \), \( p_j^*(\omega) \). Therefore, at equilibrium we can write \( \pi_i \) as a function of \( \omega \) as follows:

\[
\pi_A(p_A^*(\omega), p_B^*(\omega), x_A^*(\omega), x_B^*(\omega)).
\]

Note that \( \pi_A \) is the operating profit, and therefore \( \partial \pi_A / \partial \omega = 0 \). We have

\[
\frac{d\pi_A}{d\omega} = \left( \frac{\partial \pi_A}{\partial p_A} \frac{dp_A^*}{d\omega} + \frac{\partial \pi_A}{\partial x_A} \frac{dx_A^*}{d\omega} \right) + \left( \frac{\partial \pi_A}{\partial p_B} \frac{dp_B^*}{d\omega} + \frac{\partial \pi_A}{\partial x_B} \frac{dx_B^*}{d\omega} \right) \leq 0,
\]

if \( d\pi_A / d\omega < 0 \), then \( \omega = 0 \). If the direct effect is negative, then the incentives to acquire minority shareholdings depend on the sign of the strategic effect.

Consider the first-order conditions of the second-period:

\[
\frac{\partial \pi_A}{\partial p_A} + \omega \frac{\partial \pi_B}{\partial p_A} = 0.
\]
Since $\partial \pi_B / \partial p_A > 0$, it is clear that $\partial \pi_A / \partial p_A < 0$. Clearly, $\partial \pi_A / \partial p_B > 0$. Note also that, in the first period for $\pi_i(x_i, x_j, p_i^*, p_j^*)$, the first-order condition is

$$\frac{\partial \pi_A}{\partial x_A} + \omega \frac{\partial \pi_B}{\partial x_A} = 0,$$

since the game is tough with respect to R&D investments: $\partial \pi_B / \partial x_A < 0$, necessarily at equilibrium $\partial \pi_A / \partial x_A > 0\).\footnote{In particular, for $\Delta_i = \omega = 0$, $\partial \pi_A / \partial x_A = \rho/6 > 0.$}

For $t$ sufficiently large, evaluated at $\omega = 0$, we get

$$\text{sign} \left\{ \frac{dp^*_A}{d\omega} \bigg|_{\omega=0} \right\} > 0, \quad \text{sign} \left\{ \frac{dp^*_B}{d\omega} \bigg|_{\omega=0} \right\} > 0,$$

$$\text{sign} \left\{ \frac{dx^*_A}{d\omega} \bigg|_{\omega=0} \right\} < 0, \quad \text{sign} \left\{ \frac{dx^*_B}{d\omega} \bigg|_{\omega=0} \right\} > 0 \quad \text{and} \quad \text{sign} \left\{ \frac{\partial \pi_A}{\partial x_B} \bigg|_{\omega=0} \right\} < 0.$$

Therefore, the direct effect is negative (as in the standard game), whereas the strategic effect has one positive component and one negative component. For $t$ sufficiently large:

$$\frac{d\pi_A}{d\omega} \bigg|_{\omega=0} = \left( \frac{\partial \pi_A}{\partial p_A} \frac{dp_A^*}{d\omega} + \frac{\partial \pi_A}{\partial x_A} \frac{dx_A^*}{d\omega} \right) + \left( \frac{\partial \pi_A}{\partial p_B} \frac{dp_B^*}{d\omega} + \frac{\partial \pi_A}{\partial x_B} \frac{dx_B^*}{d\omega} \right) \leq 0.$$

While the R&D investment choices are strategic substitutes and therefore introduce a negative strategic effect, price choices are strategic complements and therefore introduce a positive strategic effect. This positive strategic effect, $(\partial \pi_A / \partial p_B) (dp_B^*/d\omega)$, can make PPO acquisition profitable.

The intuition is as follows. At period 2, for some given $x_i, x_j$, a higher $\omega$ softens the competitive pressure of firm $A$, which results in a higher $p_A$. Since price choices are strategic complements, firm $B$ also increases its price: $p_B$. Specifically,

$$\frac{\partial p_A}{\partial \omega} = \frac{2}{3\omega^2} \left( 3 - \Delta_A \right) - \frac{\rho (x_A - x_B)}{(3 - \omega)^2} = 2 \frac{\partial p_B}{\partial \omega}.$$

That is, the increase in $p_A$ is twice the increase in $p_B$. Since R&D investments make firm $A$ tough ($\partial \pi_B / \partial x_A < 0$) and prices are strategic complements, at period 1 firm $A$ adopts a puppy dog strategy: $A$ under-invests so as not to trigger an aggressive response from its rival. A higher $\omega$ makes the second-period game less tough since prices increase with $\omega$, but since the increase in $p_B$ is half of the increase in $p_A$, firm $A$ under-invests even more to reduce the competitive pressure in period 2. Thus, $x_A$ decreases and because of R&D investments are strategic substitutes, $x_B$ increases. The impact of the changes of the choice variables on $A$’s profit is intuitively clear:

- The increase in $p_A$ decreases $\pi_A$ because $A$ is overpricing (recall that $A$ maximizes $\pi_A + \omega \pi_B$)
• The decrease in $x_A$ decreases $\pi_A$ because $A$ is under-investing (because of the tough nature of the subgame)

• The increase in $p_B$ increases $\pi_A$ because for the same price $p_A$, firm $A$ obtains a higher market share

• The increase in $x_B$ decreases $\pi_A$ because it has a negative direct effect on $A$’ market share, and moreover it makes competition more tough in the second period

Therefore, only the positive strategic effect of $\omega$ on rival price can induce $A$ to acquire some financial interests in firm $B$.

6 Concluding Remarks

This paper has studied how asymmetries in passive partial ownership (PPO) holdings or asymmetries in initial competitive advantage affect R&D investments, competition, and welfare. We build a two-period model with two firms, such that in the first period firms invest in R&D to increase product quality, and in the second period firms compete in prices à la Hotelling for the demand of a differentiated product that is purchased by continuum of consumers that buy from either of the firms. We characterize the Subgame Perfect Equilibrium of the game, and we consider the second-best welfare benchmark. Our model applies to markets in which rival firms have PPO holdings for purely financial reasons, and in which PPO holdings are small so that firms do not obtain control rights. In addition, our model is relevant in markets where R&D spillovers are negligible due to high patent protection.

In contrast to previous findings, we find that PPO can increase total surplus in markets with competition in prices and quality-enhancing R&D with no spillovers. This is due to asymmetries in either PPO holdings between rivals, or due to one of the firms enjoying a sufficiently large initial competitive advantage. The target firm or the firm with the initial competitive advantage over-invest in R&D in relation to when there are no PPO holdings, leading to a higher market share, which means that a larger proportion of consumers buy a good of higher quality. This leads to greater aggregate utility than if there were no PPO holdings. If consumers give sufficient high value to quality in relation to cost of R&D and transport then total surplus will be larger.

Our finding suggests that competition authorities should take into account the potential beneficial effects of asymmetric PPO holdings in markets with competition in prices, and quality-enhancing R&D even if there are no spillovers.

Our paper suggests a few open questions for future research. Future work could study the welfare effects of asymmetries in active partial ownership stakes. Furthermore, the paper calls for the development of further empirical work which investigates the role of asymmetries (either in PPO stakes or in initial competitive advantage) on prices and R&D investments.
References


7 Appendix

The following appendix provides a proof of the propositions in the text.
Proof of Proposition 1 We start solving backwards and use the first order condition at \( t = 2 \): \( \frac{\partial \pi_i}{\partial p_i} = 0 \) for \( i = A, B \). Hence, the reaction functions for for firms \( i, j = A, B \) and \( i \neq j \) are

\[
p_i(\omega; \omega) = \frac{(1 - \omega)\Delta_i + (1 - \omega)p(x_i - x_j) + p_j + (1 - \omega)t}{2(1 - \omega)}.
\]

Finding the fixed point we note that

\[
p_i(\omega; \omega) = \frac{(1 - \omega)}{2(\omega - 3)(2\omega - 1)}(\Delta_i(1 - 2\omega) + t(3 - 2\omega) + \rho(x_i - x_j)(1 - 2\omega)),
\]

with \( s_i = \frac{1}{2t}(\Delta_i + \rho(x_i - x_j) + (p_j - p_i) + t) \) and \( p_j - p_i = -2(\Delta_i + \rho(x_i - x_j)) \frac{1 - \omega}{3 - 2\omega} \).

Substituting these expressions into profits for each firm, we can then solve at \( t = 1 \) for the optimal amount of R&D investment: \( \frac{\partial \pi_i}{\partial x_i} = 0 \) for \( i, j = A, B \) and \( i \neq j \) such that the second order condition, \( \lambda t(2\omega - 3)^2 - \rho^2 > 0 \) is satisfied and that the stability condition requires that

\[
\frac{\partial^2 \pi_A}{\partial x_A^2} \frac{\partial^2 \pi_B}{\partial x_B^2} > \frac{\partial^2 \pi_A}{\partial x_A \partial x_B} \frac{\partial^2 \pi_B}{\partial x_B \partial x_A},
\]

which is equivalent that \( \lambda t(2\omega - 3)^2 - 2\rho^2 > 0 \) applies. From the first order condition, we obtain

\[
x_i(\omega; \omega) = \frac{-\rho t(1 - \omega)(2\omega - 3) + \rho \Delta_i}{\lambda t(2\omega - 3)^2 - \rho^2} - \frac{\rho^2(1 - \omega)}{(1 - \omega)(\lambda t(2\omega - 3)^2 - \rho^2)} x_i.
\]

Hence, finding the fixed point, find the results stated in Proposition 2.

Proof of Corollary A Let us draw comparative statics of the equilibrium allocation with symmetric PPO with respect to \( \omega \).

\[
\frac{\partial s^*(\omega)}{\partial \omega} = \frac{\lambda \Delta_i}{t (\lambda (2\omega - 3)^2 - 2\rho^2)^2}
\]

and

\[
\frac{\partial p_i^*(\omega)}{\partial \omega} = \frac{t (\lambda (2\omega - 3)^2 - 2\rho^2)^2 - \Delta_i \lambda (2\omega - 1)^2 (\lambda (2\omega - 3)^2 - 2\rho^2(5 - 4\omega))}{(1 - 2\omega)^2 (t \lambda (2\omega - 3)^2 - 2\rho^2)^2},
\]

and

\[
\frac{\partial x_i^*(\omega)}{\partial \omega} = -\rho \frac{(9t\lambda - 2\rho^2 - 12t \lambda \omega + 4t \lambda \omega^2)^2 - 4t \lambda^2 \Delta_i (3 - 2\omega)^3}{\lambda (2\omega - 3)^2 (-4t \lambda \omega^2 + 12t \lambda \omega + 2\rho^2 - 9t\lambda)^2}.
\]

Let us consider several cases:

- If \( \Delta_i = 0 \) then \( \frac{\partial s^*(\omega)}{\partial \omega} = 0, \frac{\partial p_i^*(\omega)}{\partial \omega} > 0, \frac{\partial x_i^*(\omega)}{\partial \omega} < 0. \)

- If \( \Delta_i < 0 \) then \( \frac{\partial s^*(\omega)}{\partial \omega} < 0, \frac{\partial p_i^*(\omega)}{\partial \omega} < 0, \) and

\[
\text{sign}(\frac{\partial p_i^*(\omega)}{\partial \omega}) = \text{sign}((t \lambda (2\omega - 3)^2 - 2\rho^2)^2 - \Delta_i \lambda (2\omega - 1)^2 (t \lambda (2\omega - 3)^2 - 2\rho^2(5 - 4\omega))).
\]
• If \( \Delta_i > 0 \) then \( \frac{\partial x_i^*(\omega)}{\partial \omega} > 0 \), and

\[
\text{sign}\left(\frac{\partial x_i^*(\omega)}{\partial \omega}\right) = \text{sign}(4t\lambda^2 \Delta_i (3 - 2\omega)^3 - (9t\lambda - 2\rho^2 - 12t\lambda\omega + 4t\lambda\omega^2)^2),
\]

\[
\text{sign}\left(\frac{\partial p_i^*(\omega)}{\partial \omega}\right) = \text{sign}((t\lambda (2\omega - 3)^2 - 2\rho^2)^2 - \Delta_i^2 (2\omega - 1)^2 (t\lambda (2\omega - 3)^2 - 2\rho^2(5 - 4\omega))).
\]

□

**Proof of Proposition 2**  The first order conditions for firm A, \( \frac{\partial \Pi_A}{\partial p_A} = 0 \), and for firm B \( \frac{\partial \Pi_B}{\partial p_B} = 0 \) imply

\[
p_A(\omega; 0) = \frac{1}{2} (\Delta_A + \rho(x_A - x_B) + p_B(1 + \omega) + t).
\]

Substituting the expression of \( p_B \) into A’s price reaction function we obtain:

\[
p_A(\omega; 0) = \frac{1}{3 - \omega} (t(3 + \omega) + (1 - \omega)(\Delta_A + \rho(x_A - x_B))),
\]

and from the earlier proposition we know that

\[
p_B(\omega; 0) = \frac{1}{3 - \omega} (3t - \Delta_A - \rho(x_A - x_B)),
\]

with \( s(\omega; 0) = \frac{1}{3(3 - \omega)} (t(3 - 2\omega) + \Delta_A + \rho(x_A - x_B)). \) Now, we can solve \( t = 1 \) optimal amount of R&D. The first order condition for A satisfies: We obtain that \( \frac{\partial \Pi_A}{\partial x_A} = 0 \) and the second order condition is satisfied if \( \lambda t(3 - \omega)^2 - \rho^2 > 0 \), and the stability condition requires that \( \lambda t(3 - \omega)^2 - 2\rho^2 > 0 \). The first order condition, implies the following reaction function:

\[
x_A(\omega; 0) = \rho\frac{\Delta_A + t(\omega^2 - 5\omega + 3) - \rho x_B}{(\lambda t (3 - \omega)^2 - \rho^2)},
\]

and the first order condition for B and reaction function are analogous to the benchmark model without PPO. Hence,

\[
x_B(\omega; 0) = \frac{3t\rho - \rho \Delta_A}{(t\lambda (\omega - 3)^2 - \rho^2)} - \frac{\rho^2 x_A}{(t\lambda (\omega - 3)^2 - \rho^2)},
\]

and hence by finding the fixed point the results of Proposition 3 follow.

□

**Proof of Corollary B**  Let us draw comparative statics of the equilibrium allocation with asymmetric PPO with respect to \( \omega \).

\[
\frac{\partial x_A^*(\omega)}{\partial \omega} = \frac{\rho}{\lambda} \frac{(9 - \omega) (3 - \omega)^3 t^2 \lambda^2 - 2 (3 - \omega) t^2 \lambda^2 \Delta_A - (7 - 2\omega) (3 - \omega)^2 \lambda \rho^2 t + 2\rho^4}{(\omega - 3)^2 \left(- t \lambda (3 - \omega)^2 + 2\rho^2 \right)^2},
\]
\[
\frac{\partial x_B^*(\omega)}{\partial \omega} = \frac{\rho}{\lambda} 2 (3 - \omega)^3 \Delta_A t^4 \lambda^2 - 6 t^2 \lambda^2 (3 - \omega)^3 + (3 - 2\omega) (3 - \omega)^2 t \lambda \rho^2 + 2 \rho^4, \\
(3 - \omega)^2 \left( -t \lambda (3 - \omega)^2 + 2 \rho^2 \right)^2
\]

\[
\frac{\partial s_A^*(\omega)}{\partial \omega} = \frac{-3 (\omega - 3)^2 t^2 \lambda^2 + t \lambda \rho^2 (\omega^2 - 4\omega - 3) + \Delta_A \lambda (2 \rho^2 + t \lambda (\omega - 3)^2) + 2 \rho^4}{2 \left( -t \lambda (3 - \omega)^2 + 2 \rho^2 \right)^2},
\]

\[
\frac{\partial p_A^*(\omega)}{\partial \omega} = t \frac{2 \rho^4 (3 - 2\omega) + 6 (\omega - 3)^2 t^2 \lambda^2 + \lambda \rho^2 t (-3\omega^2 + 26\omega - 39) + 2 \Delta_A \lambda (2 \rho^2 (2 - \omega) - (\omega - 3)^2 t \lambda)}{(-t \lambda (3 - \omega)^2 + 2 \rho^2)^2},
\]

\[
\frac{\partial p_B^*(\omega)}{\partial \omega} = -t \frac{(-3 (3 - \omega)^2 t^2 \lambda^2 + t \lambda \rho^2 (\omega^2 - 4\omega - 3) + 2 \rho^4 + \Delta_A \lambda ((3 - \omega)^2 t \lambda + 2 \rho^2))}{(-t \lambda (3 - \omega)^2 + 2 \rho^2)^2}.
\]

Let us consider several cases:

- When \( \Delta_A = 0 \) then

\[
\frac{\partial p_A^*(\omega)}{\partial \omega} = \frac{t (2t \lambda - \rho^2) (3t \lambda (\omega - 3)^2 - 2 \rho^2 (3 - 2\omega))}{(9t \lambda - 2 \rho^2 - 6t \lambda \omega + t \lambda \omega^2)^2},
\]

which means that \( \frac{\partial p_A^*(\omega)}{\partial \omega} > 0 \) and \( \frac{\partial p_B^*(\omega)}{\partial \omega} < 0 \).

For R&D investments, we observe that

\[
\frac{\partial x_A^*(\omega)}{\partial \omega} = \frac{\rho}{\lambda} \frac{(9 - \omega) (3 - \omega)^3 t^2 \lambda^2 + (7 - 2\omega) (3 - \omega)^2 \lambda t \rho^2 - 2 \rho^4}{(3 - \omega)^2 \left( -t \lambda (3 - \omega)^2 + 2 \rho^2 \right)^2},
\]

\[
\frac{\partial x_B^*(\omega)}{\partial \omega} = \frac{\rho \lambda}{6 \lambda t^2 \lambda^2 (3 - \omega)^3 - (3 - 2\omega) (3 - \omega)^2 t \lambda \rho^2 - 2 \rho^4}{(3 - \omega)^2 \left( -t \lambda (3 - \omega)^2 + 2 \rho^2 \right)^2},
\]

which means that under most circumstances \( \frac{\partial x_A^*(\omega)}{\partial \omega} < 0 \) and \( \frac{\partial x_B^*(\omega)}{\partial \omega} > 0 \).

\[
\frac{\partial s_A^*(\omega)}{\partial \omega} = \frac{-3 (\omega - 3)^2 t^2 \lambda^2 + t \lambda \rho^2 (\omega^2 - 4\omega - 3) + 2 \rho^4}{2 \left( -t \lambda (3 - \omega)^2 + 2 \rho^2 \right)^2},
\]

which implies that when \( \rho \) is high in relation to \( t \lambda \) then \( \frac{\partial s_A^*(\omega)}{\partial \omega} > 0 \). Otherwise, \( \frac{\partial s_A^*(\omega)}{\partial \omega} < 0 \).

When \( \Delta_A \neq 0 \) then
\[
sign \left( \frac{\partial p_A^* (\omega)}{\partial \omega} \right) = \text{sign} \left( 2\rho^4 (3 - 2\omega) + 6(\omega - 3)^2 t^2 \lambda + \lambda \rho^2 t(-3\omega^2 + 26\omega - 39) - 2\Delta_A \lambda (\omega - 3)^2 t \lambda - 2\rho^2 (2 - \omega) \right),
\]

\[
sign \left( \frac{\partial p_B^* (\omega)}{\partial \omega} \right) = \text{sign} \left( 3(3 - \omega)^2 t^2 \lambda^2 - t \lambda \rho^2 (\omega^2 - 4\omega - 3) - 2\rho^4 - \Delta_A \lambda ((3 - \omega)^2 t \lambda + 2\rho^2) \right),
\]

\[
sign \left( \frac{\partial s^*_A (\omega)}{\partial \omega} \right) = \text{sign} \left( -3(\omega - 3)^2 t^2 \lambda^2 + t \lambda \rho^2 (\omega^2 - 4\omega - 3) + 2\rho^4 + \Delta_A \lambda (2\rho^2 + t \lambda (\omega - 3)^2) \right).
\]

Notice that \( \text{sign} \left( \frac{\partial p_B^* (\omega)}{\partial \omega} \right) = -\text{sign} \left( \frac{\partial s^*_A (\omega)}{\partial \omega} \right), \) and

\[
\text{sign} \left( \frac{\partial x_A^* (\omega)}{\partial \omega} \right) = \text{sign} \left( -t^2 \lambda^2 (9 - \omega) (3 - \omega)^3 + (7 - 2\omega) (3 - \omega)^2 \lambda \rho^2 - 2\rho^4 \right),
\]

\[
\frac{\partial x_B^* (\omega)}{\partial \omega} = \text{sign} \left( 6t^2 \lambda^2 (3 - \omega)^3 - (3 - 2\omega) (3 - \omega)^2 t \lambda \rho^2 - 2\rho^4 \right).
\]

**Proof of Corollary 2** The comparison of prices when the second order conditions are satisfied and \( \Delta_i = 0 \) is as follows:

\[
p_i^*(0; 0) - p_B^*(\omega; 0) = t\omega \frac{(3 - \omega)t \lambda + \rho^2}{9t \lambda - 2\rho^2 - 6t \lambda \omega + t \lambda \omega^2} < 0,
\]

\[
p_i^*(\omega; \omega) - p_i^*(0; 0) = t\omega \frac{\omega}{1 - 2\omega} > 0,
\]

\[
p_i^*(\omega; \omega) - p_B^*(\omega; 0) = t\omega \frac{t \lambda (\omega + 2) (3 - \omega) - \rho^2 (3 - 2\omega)}{(1 - 2\omega)(9 t \lambda - 2 \rho^2 - 6 t \lambda \omega + t \lambda \omega^2)},
\]

\[
p_i^*(\omega; \omega) - p_A^*(\omega; 0) = t\omega \frac{t \lambda (3\omega + 1) (3 - \omega) + \rho^2 (2\omega^2 - 7\omega + 1)}{(1 - 2\omega)(9t \lambda - 2 \rho^2 - 6t \lambda \omega + t \lambda \omega^2)},
\]

\[
p_A^*(\omega; 0) - p_i^*(0; 0) = t\omega (3 - \omega) \frac{2t \lambda - \rho^2}{9t \lambda - 2 \rho^2 - 6t \lambda \omega + t \lambda \omega^2},
\]

\[
p_A^*(\omega; 0) - p_A^*(\omega; 0) = t\omega \frac{t \lambda (3 - \omega) - \rho^2 (4 - \omega)}{9t \lambda - 2 \rho^2 - 6t \lambda \omega + t \lambda \omega^2},
\]

and the results from the Corollary follow since we also apply the second order conditions. Let us now analyze the investment in R&D.
Then, we note that:

\[
\text{sign}(x_A^*(\omega; \omega) - x_A^*(0; 0)) = \text{sign} \left( \Delta_i - \frac{(9t\lambda - 2\rho^2) \left( t\lambda (2\omega - 3)^2 - 2\rho^2 \right)}{12t\lambda^2 (3 - 2\omega) (3 - \omega)} \right),
\]

\[
\text{sign}(x_A^*(\omega; 0) - x_B^*(\omega; 0)) = \text{sign} \left( \Delta_A - \frac{t\omega (5 - \omega)}{2} \right),
\]

\[
\text{sign}(x_A^*(\omega; 0) - x_A^*(0; 0)) = \text{sign} \left( \Delta_A - \frac{(3 - \omega) (9 - 2\omega) t\lambda - \rho^2) (9t\lambda - 2\rho^2)}{3t\lambda^2 (3 - \omega) (6 - \omega)} \right),
\]

and \( \frac{(3 - \omega)(9 - 2\omega)t\lambda - \rho^2)(9t\lambda - 2\rho^2)}{3t\lambda^2(3 - \omega)(6 - \omega)} > 0 \) is positive. Furthermore,

\[
\text{sign}(x_B^*(\omega; 0) - x_B^*(0; 0)) = \text{sign} \left( \frac{(9t\lambda - 2\rho^2) (t\lambda (3 - \omega) (6 - \omega) + \rho^2)}{3t\lambda^2 (\omega - 3) (\omega - 6)} - \Delta_A \right),
\]

\[
\text{sign}(x_A^*(\omega; 0) - x_A^*(\omega; \omega)) = -\text{sign} \left( (3 - \omega) (\omega^2 - 6\omega + 6) t\lambda + \rho^2 + \frac{3t\lambda^2 \Delta_A (2 - \omega) (3 - \omega) (3 - 2\omega)}{(t\lambda (2\omega - 3)^2 - 2\rho^2)} \right).
\]

Then, we note that:

\[
\text{sign}(x_B^*(\omega; 0) - x_B^*(\omega; \omega)) = \text{sign} \left( \frac{\Delta_A^+ + (2\rho^2 + t\lambda^2 (3 - \omega) (\omega^2 - 7\omega + 9) (2\omega - 3)^2 + t\lambda^2 (2\omega^3 - 24\omega^2 + 72\omega - 63))}{3t\lambda^2 (2\omega - 3) (\omega - 6)} \right).
\]

Notice that when \( \Delta_A = 0 \) then \( x_B^*(\omega; 0) - x_B^*(\omega; \omega) = \omega \rho \frac{(t\lambda (3 - \omega) (\omega^2 - 7\omega + 9) - \rho^2)}{\lambda (3 - 2\omega) (3 - 2\omega) (3 - 2\omega)} > 0. \)

For the combined R&D of both firms, we notice that for all \( \Delta_i \): \( X^*(\omega; \omega) - X^*(0; 0) = -\frac{2\omega}{\lambda (3 - 2\omega)} < 0, \) \( X^*(\omega; 0) - X^*(\omega; \omega) = \frac{\omega \rho}{\lambda (3 - 2\omega)(3 - 2\omega)} > 0, \) and \( X^*(\omega; 0) - X^*(0; 0) = \frac{-\omega \rho}{3\lambda (3 - 2\omega)} < 0. \)

**Proof of proposition 3** Let us first assume that \( \Delta_i = 0 \) then \( x_0^A = x_0^B \) and

\[
TS^*(0; 0) = x_0^B + \left( \frac{2\rho^2}{9\lambda} - \frac{t}{4} \right),
\]

\[
TS^*(\omega; \omega) = x_0^B + \frac{\rho^2 (1 - \omega)}{\lambda (3 - 2\omega)} \left( \frac{2 - \omega}{3 - 2\omega} \right) - \frac{t}{4}.
\]
\[ TS^*(\omega; 0) = x_0^B + \\
\quad + \frac{-t^3 \lambda^3 (2 \omega^2 - 6 \omega + 9) (3 - \omega)^4 + t \lambda \rho^4 (-2 \omega^3 + 5 \omega^2 + 24 \omega - 36) (\omega - 3)^2}{4 \lambda (3 - \omega)^2 (t \lambda (3 - \omega)^2 - 2 \rho^2)^2} \\
\quad + \frac{4 \rho^6 (2 - \omega) (4 - \omega) + 2t^2 \lambda^2 \rho^2 (\omega^4 - 8 \omega^3 + 28 \omega^2 - 63 \omega + 54) (\omega - 3)^2}{4 \lambda (3 - \omega)^2 (t \lambda (3 - \omega)^2 - 2 \rho^2)^2}. \]

Now, let us compute the differences when \( \Delta_i = 0 \). We first note that

\[ TS^*(\omega; \omega) - TS^*(0; 0) = \frac{-\omega \rho^2 (3 - \omega)}{9 \lambda (2 \omega - 3)^2} < 0, \]

in fact note that \( \frac{\partial TS^*(\omega; \omega)}{\partial \omega} = \frac{-\rho^2}{\lambda (3 - 2 \omega)^3} < 0. \) Hence,

\[ TS^*(\omega; 0) - TS^*(0; 0) = \omega \frac{-4 \rho^6 (6 - \omega) + t \lambda \rho^4 (18 \omega^2 - 77 \omega - 24) (\omega - 3)^2}{36 \lambda (\omega - 3)^2 (9t \lambda - 2 \rho^2 - 6t \lambda \omega + t \lambda \omega^2)^2} \]

\[ + \frac{2t^2 \lambda^2 \rho^2 (18 \omega - 24 \omega^2 + 5 \omega^3 - 27) (\omega - 3)^2 - 9t^3 \lambda^3 \omega (\omega - 3)^4}{36 \lambda (\omega - 3)^2 (9t \lambda - 2 \rho^2 - 6t \lambda \omega + t \lambda \omega^2)^2}. \]

Hence,

\[ \text{sign}(TS^*(\omega; 0) - TS^*(0; 0)) = \]

\[ \text{sign}(-4 \rho^6 (6 - \omega) - t \lambda \rho^4 (18 \omega^2 - 77 \omega - 24) (\omega - 3)^2 \]

\[ + 2t^2 \lambda^2 \rho^2 (5 \omega^3 - 24 \omega^2 + 18 \omega - 27) (\omega - 3)^2 - 9t^3 \lambda^3 \omega (\omega - 3)^4). \]

The first, third and fourth terms are negative for the range of \( \omega \) considered. The second term is positive for the range of \( \omega \) considered. Hence when \( \rho \) is large in relation to \( t \lambda \) then total surplus with asymmetric PPO may be larger than with no PPO, and hence also than with symmetric PPO.

Another comparison is as follows:

\[ TS^*(\omega; 0) - TS^*(\omega; \omega) = \]

\[ \omega \frac{12 \rho^6 (2 - \omega) - t \lambda \rho^4 (-93 \omega + 126 \omega^2 - 60 \omega^3 + 8 \omega^4 + 24) (\omega - 3)^2}{4 \lambda (2 \omega - 3)^2 (\omega - 3)^2 (9t \lambda - 2 \rho^2 - 6t \lambda \omega + t \lambda \omega^2)^2} \]

\[ + \omega \frac{2t^2 \lambda^2 \rho^2 (-36 \omega + 25 \omega^3 - 14 \omega^4 + 2 \omega^5 + 27) (\omega - 3)^2 - t^3 \lambda^3 \omega (2 \omega - 3)^2 (\omega - 3)^4}{4 \lambda (2 \omega - 3)^2 (\omega - 3)^2 (9t \lambda - 2 \rho^2 - 6t \lambda \omega + t \lambda \omega^2)^2}. \]

Note that the first, second and third terms are positive while the fourth term is negative. Then, if \( \rho \) is small in relation to \( t \lambda \) are large then the last term could potentially dominate the previous two terms. Hence, if some conditions are satisfied then total surplus with asymmetric PPO may be smaller than with symmetric PPO.
Proof of Corollary 2. Let us first assume that $\Delta_i \neq 0$ then

$$TS^*(0;0) = \frac{1}{2} \left( (x_0^A + x_0^B) + \frac{3\lambda \Delta_A^2}{9\lambda - 2\rho^2} \right) + \left( 1 + \frac{9\lambda^2\Delta_A^2}{(9\lambda - 2\rho^2)^2} \right) \left( \frac{2\rho^2 - t}{9\lambda - 4} \right),$$

$$TS^*(\omega;\omega) = \frac{(x_0^A + x_0^B)}{2} + \rho^2 \frac{(1-\omega)}{\lambda (3-2\omega)} \left( \frac{2-\omega}{3-2\omega} \right) - \frac{t}{4} + \frac{\lambda^2 \Delta_A^2 (2\omega-3)^2}{(t\lambda (2\omega-3)^2-2\rho^2)^2} \left( \frac{2\rho^2 (1-\omega)}{\lambda (2\omega-3)^2} - \frac{t}{4} \right) + \frac{\lambda \Delta_A^2 (3-2\omega)}{2(t\lambda (2\omega-3)^2-2\rho^2)^2}.$$

Total surplus with asymmetric PPO is beyond the scope of this paper. The comparison of the previous two suggests that

$$TS^*(\omega;\omega) - TS^*(0;0) = -\frac{\omega \rho^2 (3-\omega)}{9\lambda (2\omega-3)^2} + \frac{\omega \Delta_A^2 t\lambda^2 (4t\lambda \rho^2 (4\omega^3 + 12\omega^2 - 54\omega + 27))}{(9t\lambda - 2\rho^2)^2 (t\lambda (2\omega-3)^2 - 2\rho^2)^2} + \frac{\omega \Delta_A^2 t\lambda^2 (9t^2 \lambda^2 (6-5\omega)(2\omega-3)^2 - 4\rho^4 (4\omega^2 - 13\omega + 12))}{(9t\lambda - 2\rho^2)^2 (t\lambda (2\omega-3)^2 - 2\rho^2)^2},$$

it sign can be re-written as

$$\text{sign}(TS^*(\omega;\omega) - TS^*(0;0)) = \text{sign} \left( \Delta_A^2 \chi - \zeta \right)$$

where $\chi = \left( 4t\lambda \rho^2 (4\omega^3 + 12\omega^2 - 54\omega + 27) + 9t^2 \lambda^2 (6-5\omega)(2\omega-3)^2 - 4\rho^4 (4\omega^2 - 13\omega + 12) \right)$ and $\zeta = \frac{\rho^2 (3-\omega) (9t\lambda - 2\rho^2)^2 (t\lambda (2\omega-3)^2 - 2\rho^2)^2}{9t\lambda (2\omega-3)^2}. \blacksquare$

Proof of Proposition 4. i) Symmetric PPO

We first need to calculate the second-best welfare optimal R&D investment taking as given the equilibrium prices at $t=2$. As such, we have that $s = \frac{-t(2\omega-3) + \Delta_A + p(x_A - x_B)}{2t(3-2\omega)}$, and hence $\frac{\partial s(\omega)}{\partial x_A} = \frac{-\rho}{2t(2\omega-3)}$ and $\frac{\partial s(\omega)}{\partial x_B} = \frac{\rho}{2t(2\omega-3)}$. The first order condition is then

$$\frac{\partial TS}{\partial x_A} = \frac{\partial s}{\partial x_A} \Delta_A + p(x_A - x_B) \frac{\partial s}{\partial x_A} + ps - \frac{t}{2} (2s - 2(1-s)) \frac{\partial s}{\partial x_A} - \lambda x_A,$$

and the second order condition is

$$\frac{\partial^2 TS}{\partial x_A^2} = \frac{\rho^2 (5-4\omega) - 2t\lambda (2\omega-3)^2}{2t (2\omega-3)^2} < 0,$$

which is equivalent to $\rho^2 < \frac{2t\lambda (2\omega-3)^2}{(5-4\omega)}$ and note that $\frac{\partial^2 TS}{\partial x_A \partial x_B} = \frac{-\rho^2 (5-4\omega)}{2t(2\omega-3)^2} < 0$. If the stability condition $\lambda t (2\omega-3)^2 - \rho^2 (5 - 4\omega) > 0$. Hence, we obtain the reaction functions in the first period.
\[ x_i = \frac{\rho \Delta_1 (5 - 4\omega) + \rho t (2\omega - 3)^2}{2t\lambda (2\omega - 3)^2 - \rho^2 (5 - 4\omega)} x_j. \]

Then, finding the fixed point we obtain the second-best efficient R&D investments:

\[ x_i^{**}(\omega; \omega) = \frac{\rho}{2\lambda} \left( 1 + \frac{\lambda \Delta_1 (5 - 4\omega)}{(t\lambda (2\omega - 3)^2 - \rho^2 (5 - 4\omega))} \right) \]

As a special case, we find that with no PPO

\[ x_i^{**}(0; 0) = \frac{\rho}{2\lambda} \left( 1 + \frac{\lambda \Delta_A}{(9t\lambda - \rho^2)} \right) \]

ii) Comparison of the equilibrium and efficient R&D investments.
Comparing the expression of the second best optimal R&D and expression with the equilibrium one, we obtain that \( x^*(\omega; \omega) - x_i^{**}(\omega; \omega) = \rho \frac{-t^2 \lambda^2 (2\omega - 3)^4 - 2t^4 (5 - 4\omega) + t \lambda \rho^2 (2\omega - 3)^2 - t \lambda^2 \Delta_1 (3 - 4\omega) (3 - 2\omega)^3}{2\lambda (3 - 2\omega) (3 - 2\omega)^3} \), which implies

\[ \text{sign}(x_i^*(\omega; \omega) - x_i^{**}(\omega; \omega)) = -\text{sign}(\Delta_i + \Delta_i) \]

where \( \Delta_i^s = \frac{(t\lambda (2\omega - 3)^2 - \rho^2 (5 - 4\omega))}{t\lambda (3 - 2\omega)^3} > 0. \)

With no-cross ownership we obtain that \( x_i^*(0; 0) - x_i^{**}(0; 0) = \rho \frac{10t^3 + 63t \lambda \rho^2 - 81t \lambda^2 (\Delta_i + \Delta)}{2\lambda (3 - 2\omega)^3 (9\lambda - 5\rho^2)(9\lambda - 2\rho^2)} \), which implies

\[ \text{sign}(x_i^*(0; 0) - x_i^{**}(0; 0)) = -\text{sign}(\Delta_i^0 + \Delta_i), \]

where \( \Delta_i^0 = \frac{(9\lambda - 5\rho^2)(9\lambda - 2\rho^2)}{81t \lambda^2} > 0. \)

We also notice that do comparative statics of the second-best optimal quantity with respect to \( \omega \)

\[ \text{sign}(\frac{\partial x_i^{**}(\omega; \omega)}{\partial \omega}) = \text{sign}(\Delta_i). \]

This implies that when \( \Delta_i > 0 \) then increasing PPO increases the amount of efficient R&D investment.

iii) Asymmetric PPO. We first find the efficient R&D investment under asymmetric PPO. Note that

\[ s = \frac{1}{2t(3 - \omega)} (t(3 - 2\omega) + \Delta_A + \rho (x_A - x_B)) \]

and hence \( \frac{\partial s(\omega)}{\partial x_A} = \frac{\rho}{2t(3 - \omega)} \) and \( \frac{\partial s(\omega)}{\partial x_B} = -\frac{\rho}{2t(3 - \omega)} \). The first order condition is then

\[ \frac{\partial TS}{\partial x_A} = \frac{\partial s}{\partial x_A} \Delta_A + \rho (x_A - x_B) \frac{\partial s}{\partial x_A} + \rho s - \frac{t}{2} (2s - 2(1 - s)) \frac{\partial s}{\partial x_A} - \lambda x_A \]
Then, substituting one into the other, we obtain the efficient R&D investments with asymmetric PPO.

\[
\frac{\partial T S}{\partial x_A} = \frac{\rho \Delta_A (5 - 2\omega) + \frac{\rho^2}{2}(x_A - x_B)(5 - 2\omega) + \rho t (2\omega^2 - 8\omega + 9) - 2t \lambda x_A (3 - \omega)^2}{2t (3 - \omega)^2}
\]

the second order condition is satisfied if

\[
\frac{\partial^2 T S}{\partial x_A^2} = \frac{\rho^2 (5 - 2\omega) - 2t \lambda (3 - \omega)^2}{2t (3 - \omega)^2} < 0
\]

and the stability condition requires that \(t \lambda (3 - \omega)^2 - \rho^2 (5 - 2\omega) > 0\). Note that \(\frac{\partial^2 T S}{\partial x_A \partial x_B} = -\frac{\rho^2 (5 - 2\omega)}{2t(3 - \omega)^2} < 0\). Hence, the best reaction function is

\[
x_A = \frac{\rho \Delta_A (5 - 2\omega) + \rho t (2\omega^2 - 8\omega + 9)}{(2t \lambda (3 - \omega)^2 - \rho^2 (5 - 2\omega))} - \frac{\rho^2 (5 - 2\omega)}{(2t \lambda (3 - \omega)^2 - \rho^2 (5 - 2\omega))} x_B
\]

For the acquired firm we obtain the following

\[
\frac{\partial T S}{\partial x_B} = -\frac{\rho \Delta_A (5 - 2\omega)}{2t (3 - \omega)^2} - \frac{\rho^2 (x_A - x_B)(5 - 2\omega)}{2t (3 - \omega)^2} + \frac{\rho t (9 - 4\omega)}{2t (3 - \omega)^2} - \frac{2t \lambda (3 - \omega)^2 x_B}{2t (3 - \omega)^2}
\]

\[
\frac{\partial^2 T S}{\partial x_B^2} = \frac{\rho^2 (5 - 2\omega) - 2t \lambda (3 - \omega)^2}{2t (3 - \omega)^2} < 0
\]

Then, from the first order condition, we obtain the reaction function

\[
x_B = -\frac{\rho \Delta_A (5 - 2\omega) + \rho t (9 - 4\omega)}{2t \lambda (3 - \omega)^2 - \rho^2 (5 - 2\omega)} - \frac{\rho^2 (5 - 2\omega)}{2t \lambda (3 - \omega)^2 - \rho^2 (5 - 2\omega)} x_A
\]

Then, substituting one into the other, we obtain the efficient R&D investments with asymmetric PPO.

\[
x_{A**}^*(\omega; 0) = \frac{(5 - 2\omega)(\lambda \Delta_A - \rho^2)}{2 \lambda (t \lambda (3 - \omega)^2 - \rho^2 (5 - 2\omega))} + \frac{t \lambda (2\omega^2 - 8\omega + 9)}{2t (3 - \omega)^2 - \rho^2 (5 - 2\omega)}
\]

\[
x_{B**}^*(\omega; 0) = \frac{-(\lambda \Delta_A + \rho^2)(5 - 2\omega) + \rho t (9 - 4\omega)}{2 \lambda \lambda (3 - \omega)^2 - \rho^2 (5 - 2\omega)}
\]

The comparison leads to

\[
sign(x_A^*(\omega; 0) - x_{A**}^*(\omega; 0)) = sign(\Delta_A^* - \Delta_A)
\]

where \(\Delta_A^* = -\frac{t \lambda^2 (2\omega + 3)(3 - \omega)^3 + t \lambda \rho^2 (\omega + 1)(7 - 2\omega)(3 - \omega)^2 - 2t \rho^4 (5 - 2\omega)}{t \lambda^2 (3 - 2\omega)(3 - \omega)}\) and

\[
sign(x_B^*(\omega; 0) - x_{B**}^*(\omega; 0)) = -sign(\Delta_B + \Delta_B^*)
\]
where \( \Delta_B^a = \frac{t^2 \lambda^2 (3-4 \omega)(3-\omega)^3+2 \rho^2 (5-2 \omega)-t \lambda \rho^2 (1-\omega)(7-2 \omega)(3-\omega)^2}{t \lambda^2 (3-2 \omega)(3-\omega)^3} \).

Let us study the derivative of these two expressions with respect to \( \omega \).

\[
\frac{\partial x_A^{***}(\omega; 0)}{d \omega} = \frac{t \rho (\rho^2 (\omega^2 - 5 \omega + 5) + \lambda \Delta_A (2 - \omega) (3 - \omega) - t \lambda (3 - 2 \omega) (3 - \omega))}{(t \lambda (3 - \omega)^2 - \rho^2 (5 - 2 \omega))^2}
\]

and

\[
\frac{\partial x_B^{***}(\omega; 0)}{d \omega} = -t \rho (\rho^2 (\omega^2 - 5 \omega + 5) + \lambda \Delta_A (2 - \omega) (3 - \omega) - t \lambda (3 - 2 \omega) (3 - \omega))}{(t \lambda (3 - \omega)^2 - \rho^2 (5 - 2 \omega))^2}
\]

Hence,

\[
\text{sign} \left( \frac{\partial x_A^{***}(\omega; 0)}{d \omega} \right) = -\text{sign} \left( \frac{\partial x_B^{***}(\omega; 0)}{d \omega} \right) = \text{sign} (\rho^2 (\omega^2 - 5 \omega + 5) - t \lambda (3 - 2 \omega) (3 - \omega) + \lambda \Delta_A (2 - \omega) (3 - \omega))
\]
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